A study of the asymptotic character of the geostrophic wind

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The dynamics of large scale atmospheric flows at midlatitudes is governed by the momentum equations

\[
\begin{align*}
\frac{du}{dt} - \frac{uv \tan \phi}{r_e} - 2\Omega v \sin \phi &= -\frac{1}{\rho} \frac{\partial p}{\partial x}, \\
\frac{dv}{dt} + \frac{u^2 \tan \phi}{r_e} + 2\Omega u \sin \phi &= -\frac{1}{\rho} \frac{\partial p}{\partial y}
\end{align*}
\]

where \( \frac{d}{dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \), together with the energy and mass conservation equations [1]. An approximate solution of Eqs. (1) is obtained by a scale analysis which leads to the well-known geostrophic wind \( \mathbf{v}_g = u_g \mathbf{i} + v_g \mathbf{j} \) \( = \mathbf{k} \times \rho^{-1} f^{-1} \nabla p \) \( (f = 2\Omega \sin \phi) \). Several authors have studied the asymptotic character of \( \mathbf{v}_g \) in order to obtain, e.g., the proper initial conditions that should be used when \( \mathbf{v}_g \) is used into (1) for short-range synoptic forecasts [2]. In fact, as is know [2,3], asymptotic solutions of (2) are valid on a time interval \( [\delta, t] \) with \( \delta > 0 \). The purpose of this work is to study the asymptotic behavior of \( \mathbf{v}_g \) by means of an alternative way.

An approximate solution \( \mathbf{v}_a = u_a \mathbf{i} + v_a \mathbf{j} \) of Eqs. (1) was found by perturbation methods [3]. In a complementary work presented in this meeting [4], it is shown that \( \mathbf{v}_a \) yields an accurate approximation of the inertial trajectories corresponding to the Eqs. (1) in a time interval \( [0, t_{\text{max}} \sim 35 \text{ hrs}] \). This time interval is suitable to study the asymptotic character of \( \mathbf{v}_g \) by means of \( \mathbf{v}_a \). In fact, as expected, the results reported in this work show that \( \mathbf{v}_g \) can be obtained by means of a suitable asymptotic expansion of the analytic expression of \( \mathbf{v}_a \). Numerical results obtained with analytic expressions of the pressure for tidal oscillations permit us to study the time interval where \( \mathbf{v}_g \) is a reliable approximation of Eqs. (1).

References.